Adding Commodities Futures to an Equity Portfolio: Lessons from Markowitz Portfolio Optimization

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Objectives of the Study

- Highlight benefits and fragility of simplistic Markowitz portfolio optimization with various commodities as assets
- Highlight some Key issues in Markowitz Optimization and thereby better understand the pluses and minuses of Optimized Portfolios which contain commodities as assets
- Focus on the implicit assumptions in Markowitz optimization and the potential problems of breaking those assumptions inherent in liquidity challenged assets (crisis issues)
- Look at how history can fool us into a sense of safety
- Examine the pitfalls of long only portfolios when considering commodity allocations

Why Commodities in Long Term Investment Portfolios?

- "Adding Commodity assets diversifies risk in those portfolios due to their low correlation to other assets!"
  - A quote that could be taken from various authors
- It is typically believed and loved as truth by most participants on financial news programs and as such putting commodities into a portfolio MUST add diversification.
- Let's examine this thought more carefully

Typical Correlation Matrix

(2002 – 2012 using excess returns)

Estimated Correlation Matrix	S&P500 Index	US Treasury 10-Year	Cash (USD)	Oil	Natural Gas	Gold	Copper	Corn	Wheat
S&P500 Index	1.0000	-0.6048	0.0000	0.5253	0.0599	0.1093	0.5056	0.1871	0.1985
US Treasury 10-Year	-0.6048	1.0000	0.0000	-0.3689	-0.0572	0.0628	-0.3272	-0.1299	-0.1480
Cash (USD)	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Oil	0.5253	-0.3689	0.0000	1.0000	0.1135	0.3504	0.5487	0.2605	0.2571
Natural Gas	**0.0599**	**-0.0572**	0.0000	0.1135	1.0000	0.1595	0.0462	0.1285	0.1541
Gold	0.1093	0.0628	0.0000	0.3504	0.1595	1.0000	0.4204	0.1664	0.1613
Copper	0.5056	-0.3272	0.0000	0.5487	0.0462	0.4204	1.0000	0.2109	0.2410
Corn	0.1871	-0.1299	0.0000	0.2605	0.1285	0.1664	0.2109	1.0000	0.7332
Wheat	**0.1985**	**-0.1480**	**0.0000**	**0.2571**	**0.1541**	**0.1613**	**0.2410**	0.7332	1.0000

Yes, there appears to be low correlations and this may be a sufficient condition for DIVERSIFICATION

Estimated Correlation Matrix	S&P500 Index	US Treasury 10-Year	Cash (USD)	Oil	Natural Gas	Gold	Copper	Corn	Wheat
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Inappropriate Approaches to Optimizations

- Why Inappropriate in the Title?
  - Breaking most Rules of Markowitz asset allocation
- See "True Markowitz or assumptions we break and why it matters," in the *Review of Financial Economics* for a discussion of these issues in general.
- To highlight issues in using commodities as diversifiers, we are going to create typical optimized portfolios that one may see in an MBA class exercise
  - In sample optimization
  - Forcing long only
  - Using one period's correlation matrix but adjusting volatilities and expected returns to reflect those of other periods
- Having built a portfolio seeing how it behaves when shocked

Data Periods and Choices

- Pre 2008 Crisis: 2002 through 2006
  - Returns settle down to Pre Crisis returns and volatility
  - Pre Crisis portfolio and a Crisis applied to returns
- Crisis: September 2008 to March 2009
  - Crisis occurs; if you knew what would the optimized portfolio look like
- Post Crisis:
  - Historical Returns and Volatilities (New Normal)
  - Consider a Crisis: Optimization, Optimized with New Normal but crisis occurs
  - Pre Crisis returns

Optimizations

- Three periods expected returns
- Same volatilities for each period
- Use of different expectations of returns
- 9 Assets all in excess returns
  - Three typical financial assets – Cash, S&P 500, Treasuries
  - Six Commodities: Gold, Oil, Nat gas, Copper, Corn and Wheat
- Allow for long and short positions, no constraints

Typical Data: Post Crisis

Post-Crisis		
Asset	Excess Return over LIBOR	Annualized Standard Deviation
S&P500 Index	**12.97%**	**18.88%**
US Treasury 10-Year	**4.99%**	**6.85%**
Cash (USD)	**0.00%**	**0.00%**
Oil	**8.55%**	**30.09%**
Natural Gas	**-9.13%**	**45.13%**
Gold	**18.88%**	**18.21%**
Copper	**7.48%**	**27.79%**
Corn	**26.39%**	**33.03%**
Wheat	**23.51%**	**37.97%**

Post Crisis Correlation Matrix

S&P500 Index	US Treasury 10-Year	Cash (USD)	Oil	Natural Gas	Gold	Copper	Corn	Wheat
1.0000	-0.6048	0.0154	0.5253	0.0599	0.1093	0.5056	0.1871	0.1985
-0.6048	1.0000	-0.0090	-0.3689	-0.0572	0.0628	-0.3272	-0.1299	-0.1480
0.0154	-0.0090	1.0000	0.0315	0.0058	-0.0294	0.0265	0.0420	0.0633
0.5253	-0.3689	0.0315	1.0000	0.1135	0.3504	0.5487	0.2605	0.2571
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Optimizations with Good Return Expectations – that is, we cheated

- Hold the Correlation Matrix the same
- Use Post – Crisis Volatility
- Adjust expected returns allowing for both long and short positions and borrowing or lending

Post Crisis Var-Cov Matrix: Optimizations

Assets	Portfolio Weights: Post Crisis Data	Portfolio Weights: Crisis Returns and Post Volatilities	Portfolio Weights: Pre Crisis Returns and Post Volatilities
S&P500 Index	50.01%	-52.81%	-13.33%
US Treasury 10-Year	118.11%	-128.51%	-8.83%
Cash (USD)	-88.10%	275.85%	88.86%
Oil	-5.27%	-11.77%	5.46%
Natural Gas	-3.54%	-11.02%	5.54%
Gold	26.91%	58.41%	8.40%
Copper	-9.73%	-24.70%	14.07%
Corn	10.11%	-11.48%	1.70%
Wheat	1.48%	6.03%	-1.87%
Expected Return of the Portfolio	19.62%	54.60%	8.59%
Expected Variance of the Portfolio	0.0101	0.0218	0.0034
Expected Standard Deviation of the Portfolio	10.03%	14.78%	5.86%
Expected Excess Return-Risk Ratio	**1.9560**	**3.6947**	**1.4655**
Leverage (1 or less = No Leverage)	1.88	None	None
Sum of Non-Cash Positions	188.10%	-175.00%	11.14%
Borrowings (if any)	88.10%	Placement of Cash	None

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Natural Gas	[illegible]	[illegible]	[illegible]
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Observations of Optimizations

- If one has good insight – that is we know the expected returns even if they are miserable as in most cases of the Crisis Period – and our assumptions about volatility are low enough (post crisis) then Sharpe and Information Ratios look great.
- Positions for the Pre and Post period optimizations look acceptable and fairly benign
- Positions for the Crisis expected returns with inconsistent assumptions of volatility appear extreme.
  - Positions are Large and extreme
  - Huge long cash with short positions in commodities that went down
- VOLATILITIES ASSUMPTIONS MATTER AS MUCH AS GETTING THE EXPECTED RETURNS RIGHT!
- Or, would one make these bets if volatility were more accurately known? Vols do not fall in a crisis!

COMPARING VOLATILITIES

ASSETS	PRE CRISIS VOLS	CRISIS VOLS	POST CRISIS
S&P500 Index	16.13%		18.88%
US Treasury 10-Year	6.25%		6.85%
Cash (USD)	0.09%		0.01%
Oil	34.25%		30.09%
Natural Gas	61.03%		45.13%
Gold	16.99%		18.21%
Copper	28.29%		27.79%
Corn	25.64%		33.03%
Wheat	28.99%		37.97%

COMPARING VOLATILITIES

ASSETS	PRE CRISIS VOLS	CRISIS VOLS	POST CRISIS
S&P500 Index	16.13%	57.55%	18.88%
US Treasury 10-Year	6.25%	12.02%	6.85%
Cash (USD)	0.09%	0.09%	0.01%
Oil	34.25%	95.92%	30.09%
Natural Gas	61.03%	54.74%	45.13%
Gold	16.99%	38.25%	18.21%
Copper	28.29%	69.56%	27.79%
Corn	25.64%	51.48%	33.03%
Wheat	28.99%	49.24%	37.97%

Vols typically doubled or tripled during the crisis

COMPARING VOLATILITIES

ASSETS	PRE CRISIS VOLS	CRISIS VOLS	POST CRISIS
S&P500 Index	16.13%	57.55%	18.88%
US Treasury 10-Year	6.25%	12.02%	6.85%
Cash (USD)	0.09%	0.09%	0.01%
Oil	34.25%	95.92%	30.09%
Natural Gas	**61.03%**	**54.74%**	**45.13%**
Gold	16.99%	38.25%	18.21%
Copper	28.29%	69.56%	27.79%
Corn	25.64%	51.48%	33.03%
Wheat	28.99%	49.24%	37.97%

The Exception

Commodity Risk Behavior in a Crisis

- Commodities utilized to diversify risk also create some difficulties with respect to behavior during shocks and crises.
- Thus, in building portfolios that utilize commodities for diversification which look “normal” may have hidden concerns.
  - Volatility and Correlation concerns that do not appear at first
  - Liquidity issues that exacerbate volatility movements
  - Gapping in bid-offers in a crisis may affect transactions costs or availability
- If a major issue for using commodities arises in volatility space lets examine our portfolios to better understand why “normal” may be misleading.
  - First, lets look at what the Crisis Portfolio would have looked like if we had used the vols apropos to the period.
  - Second, lets build a portfolio that is typical of most wealth managers. Lets force our allocations to be long only.
  - Third, lets shock our pre-crisis and post crisis long – short portfolios, our long only portfolios and see how they would hold up in a crisis.

Crisis Portfolio with own Vols and Expectations

Calculated Weights	
S&P500 Index	-7.71%
US Treasury 10-Year	-23.85%
Cash (USD)	135.63%
Oil	-1.29%
Natural Gas	-8.48%
Gold	13.56%
Copper	-5.34%
Corn	-5.18%
Wheat	2.65%
Expected Return of the Portfolio	13.30%
Expected Variance of the Portfolio	0.0102
Expected Standard Deviation of the Portfolio	10.11%
Expected Excess Return-Risk Ratio	1.3148
Leverage (1 or less = No Leverage)	None
Sum of Absolute Values of Exposures	203.69%
Sum of Non-Cash Positions (Shorts cancel Longs)	-35.63%
Borrowings (if any) above 100% Capital Usage	None

Realism in Risk

Calculated Weights	
S&P500 Index	-7.71%
US Treasury 10-Year	-23.85%
Cash (USD)	135.63%
Oil	-1.29%
Natural Gas	-8.48%
Gold	13.56%
Copper	-5.34%
Corn	-5.18%
Wheat	2.65%
Expected Return of the Portfolio	13.30%
Expected Variance of the Portfolio	0.0102
Expected Standard Deviation of the Portfolio	10.11%
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Leverage (1 or less = No Leverage)	None
Sum of Absolute Values of Exposures	203.69%
Sum of Non-Cash Positions (Shorts cancel Longs)	-35.63%
Borrowings (if any) above 100% Capital Usage	None

Moving toward Realism In Portfolios

- Diverge into normalcy for a moment by moving our long-short portfolios to something closer to a long-only one for Post Crisis data.
- Methodology is not to simply constrain the portfolios to have long positions only. We believe this creates corner solutions which fool investors (see *True Markowitz)*
- Thus we will find some set of return expectations (using Post Crisis volatility and variance covariance matrix) so the Optimizer will DESIRE to take long positions and avoid going short.

Excess Returns Comparison: Historical Post Crisis versus Adjusted

Assets	Long Only Adjusted	Long Short
S&P500 Index	12.00%	12.97%
US Treasury 10-Year	1.00%	4.99%
Cash (USD)	0.00%	0.00%
Oil	26.12%	8.55%
Natural Gas	33.37%	-9.13%
Gold	17.36%	18.88%
Copper	30.86%	7.48%
Corn	12.21%	26.39%
Wheat	15.00%	23.51%

Long Only Allocations

Assets	Allocation Summary	
S&P500 Index	10.91%	
US Treasury 10-Year	49.47%	
Cash (USD)	11.82%	
Oil	3.78%	
Natural Gas	5.66%	
Gold	6.56%	
Copper	11.74%	
Corn	0.10%	
Wheat	-0.05%	
Expected Return Portfolio	9.45%	
Expected Variance Portfolio	0.0038	
Expected Standard Deviation Portfolio	6.15%	
Expected Excess Return-Risk Ratio	1.5370	
Leverage (1 or less = No Leverage)	None	
Sum of Non-Cash Positions	88.18%	

Long Only Allocations

Assets	Long Only Allocations Summary – Modified expected returns	Long Short Allocation Summary – Original expected Returns
S&P500 Index	10.91%	50.01%
US Treasury 10-Year	49.47%	118.11%
Cash (USD)	11.82%	-88.10%
Oil	3.78%	-5.27%
Natural Gas	5.66%	-3.54%
Gold	6.56%	26.91%
Copper	11.74%	-9.73%
Corn	0.10%	10.11%
Wheat	0.00%	1.48%
Expected Return Portfolio	9.45%	19.62%
Expected Variance Portfolio	0.0038	0.0101
Expected Standard Deviation Portfolio	6.15%	10.03%
Expected Excess Return-Risk Ratio	1.5370	1.9560
Leverage (1 or less = No Leverage)	None	1.88
Sum of Non-Cash Positions	88.18%	188.10%

Real Issue is What Happens if we are WRONG

- We will be WRONG in our expectations!
  - Optimization exists because one is always wrong; we are only human and thus do not know the future.
  - Markowitz came up with the diversification concepts because in the real world we are "making up" our expected returns. We guess the future. No one is Prescient!
  - In (Markowitz) reality the "correct" volatilities are the errors in our forecasts of the expected returns and the correlations are those of our forecast errors not historical correlations!
- Now that we are aware that our optimization will give us portfolios that are incorrect, which ones behave better when we are REALLY WRONG since we are going to be so.
- We compare two portfolios that we created using Post Crisis Data and see what would happen if the Crisis Occurs.

Shocked Portfolios: Returns

Assets	Long – Short Optimization	Long Only Optimization
S&P500 Index	-2.87%	-4.82%
US Treasury 10-Year	0.19%	1.59%
Cash (USD)	0.00%	0.00%
Oil	0.22%	-1.86%
Natural Gas	-0.15%	-2.70%
Gold	0.88%	1.14%
Copper	0.35%	-5.69%
Corn	-0.90%	-0.04%
Wheat	-0.09%	0.00%
PORTFOLIO RETURN	-2.36%	-12.36%

Shocks: What about the Risk?

	Long –Short	Long Only
Not Scaled	27.26%	14.28%
Scaled by Expected Return	27.26%	29.55%

What About the Return Scaled for Expected Returns

	Long -Short	Long Only
Not Scaled	-2.36%	-12.36%
Scaled by Expected Return	-2.36%	-25.59%

Ten Times the Losses!

Summary Comments

- To measure the implications of putting commodities into simple portfolios many issues need to be considered.
  - How much Risk can you live with?
    - Commodities tend to have greater volatility and in crisis the Vols increase sharply
  - What target of return do you need? Do not be greedy and think commodity diversification will save the day.
  - Commodities provide diversification but when using them do not make the error of thinking they can simply be added to long only portfolio – this is particularly true if using commodities to diversify *typical* long only portfolios.
- Simple Correlation analysis can be very misleading – examine correlation stability in your forecasts.

Summary Comments

- To measure the implications of putting commodities into simple portfolios basic issues need to be considered.
  - How much Risk can you live with? – Commodity risk is not stable and one should not expect it to be so.
    - Commodities tend to have greater volatility; in crisis the Vols increase sharply -- past averages will be too low in a crisis
  - What target of return do you need? Does your need create too much risk?
  - Examine correlation behavior – if you are wrong do your correlations in errors change?
  - Simple Correlation analysis can be very misleading.
- REPEAT: Commodities provide diversification but when using them do not make the error of thinking they can simply fit into a long only portfolio –

Avoid simple *Long Only* commodity portfolios!

Know your real risk!

Drinks?